POINT EXPLOSION IN AN IDEALLY BRITTLE BODY

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 4, pp. 145–148, 1969

The formulation of the problem of a point explosion in an ideally brittle body is examined; the dependence of the characteristic dimension of the fracture zone on the intensity of the point source is established by dimensional analysis [1]. Certain physico-technical problems whose mathematical description can be reduced to the problem of a point explosion are briefly considered.

1. Formulation of the problem. In an infinite space consisting of an ideally brittle material let there be a cylindrical or spherical cavity of radius r, whose walls are acted on by a pressure p during time τ. We recall that bodies that obey a linear Hooke's law up to fracture are considered ideally brittle: in particular, near the end of a crack there is no region in which the material does not obey Hooke's law. The body is considered to be homogeneous and isotropic; it is assumed that a certain number n of initial cracks of length l_0 extend to the boundary of the cavity. At the initial instant the body is at rest.

As a result of the application of pressure the cracks develop; at sufficiently large growth rates the cracks spontaneously branch and their number increases. The fracture process ends in the cessation of "multiplication" and development of the cracks.

We denote by R a certain characteristic linear dimension of the fracture region. For example, as R we can take the maximum dimension determined by the most developed cracks or the dimension of the zone formed by unconnected pieces of material. It is very important to find this geometric characteristic as a function of the parameters of the problem.

We assume that the solution of this very complicated dynamic problem of the theory of elasticity has been found and pass to the limit

$$\tau \to 0, \quad r \to 0, \quad p \to \infty, \quad l_0/r \to 0 \tag{1.1}$$

or

$$\tau \to \infty, \quad r \to 0, \quad p \to \infty, \quad l_0/r \to 0 \tag{1.2}$$

but so that one parameter—that characterizing the intensity of the explosion—is finite. As the latter parameter, depending on the formulation of the physical problem (see below), it is possible to take either the total energy of the explosion Э or the impulse J. Obviously, these quantities have the following dimensions:

spherical case

$$[Э] = FL, \qquad [J] = FT \tag{1.3}$$

plane case

$$[Э] = F, \qquad [J] = FL^{-1}T \tag{1.4}$$

Here, F is force, L length, and T time. Thus, we obtain the problem of the fracture of an ideally brittle body under the action of a point explosion of intensity Э or J.

The explosion may take place either at the free surface of the body (surface explosion) or in the interior of the body (contained explosion): in both cases it is assumed that R is finite but much less than the characteristic linear dimension of the body, for example, the radius of curvature of the boundary at the explosion point in the first case or the distance between the center of the explosion and the surface of the body in the second.

2. Determination of the dependence of the size of the fracture region on the intensity of the explosion. The corresponding mathematical problem is formulated as follows: It is required to solve the equations of the dynamic theory of elasticity in a region with moving surfaces of discontinuity of the displacements; on the time-dependent boundary of the surfaces of discontinuity (cracks) certain additional conditions, determining the velocity and branching of the cracks, their type, and the direction of development of the crack contour, must be satisfied. The initial configuration of the cracks and the cavity in the body at rest is given.

The dynamic equations of the theory of elasticity reduce to a system of wave equations [2] in the displacement potentials (four in the general three-dimensional case and two for the plane problem). These equations contain only two parameters: c and ν, where c is the propagation velocity of the longitudinal waves and ν is Poisson's ratio ($[c] = LT^{-1}$, $[\nu] = 1$).

We assume that the free edges of the cracks are not under load and that a state of cohesion or dry friction exists at the edges in contact. In neither case does Young's modulus E enter into the boundary conditions of the limiting point problem. When dry Coulomb friction is taken into account, the boundary conditions will also contain the dimensionless coefficient of friction f of the rubbing edges of the cracks.

The additional conditions at the boundary of a dynamic crack in an ideally brittle body (determining its velocity, branching, and direction of growth) contain [3] only one new parameter $E\gamma$; here, γ is the surface energy density, a physical constant of the ideally brittle body representing the dissipation of energy due to crack growth per unit area.

Obviously, this parameter has the dimensions

$$[E\gamma] = F^2L^{-3} \tag{2.1}$$

Moreover, in the point problem we are given an additional condition determining the intensity of the explosion; it contains either the quantity ϑ or the quantity J (see (1.3) and (1.4)).

The characteristic linear dimension R of the fracture region depends only on the parameters entering into the differential equations and the initial, boundary, and auxiliary conditions of the problem.

Using the π theorem, by dimensional analysis we easily obtain the following formulas:

spherical case

$$R = \lambda_1(n, f, \nu)\frac{\vartheta^{2/5}}{(E\gamma)^{1/5}}, \qquad R = \lambda_3(n, f, \nu)\frac{(cJ)^{2/5}}{(E\gamma)^{1/5}} \tag{2.2}$$

plane case

$$R = \lambda_2(n, f, \nu)\frac{\vartheta^{2/3}}{(E\gamma)^{1/3}}, \qquad R = \lambda_4(n, f, \nu)\frac{(cJ)^{2/3}}{(E\gamma)^{1/3}} \tag{2.3}$$

Here, λ_1, λ_2, λ_3, and λ_4 are certain dimensionless functions of their arguments; from physical considerations it is natural to assume that they are approximately constant. On the left we have written the formulas for problems with finite source energy, on the right those for problems with finite impulse. From general considerations one can easily obtain certain interesting relations; the complete analytic solution of the mathematical problem is accessible only in the simplest self-similar case and for a single crack [4].

For the elastic-plastic model an analogous dimensional analysis of the point problem leads to the following relations for the size of the fracture zone corresponding to (2.2) and (2.3):

spherical case

$$\begin{aligned} R\frac{(E\gamma)^{1/5}}{\vartheta^{2/5}} &= f_1\left\{\frac{\vartheta^{1/3}}{R\sigma_s^{1/3}}, \frac{\sigma_s}{E}, \frac{\gamma}{ER}, \nu, f, n\right\} \\ R\frac{(E\gamma)^{1/5}}{(cJ)^{2/5}} &= f_2\left\{\frac{(cJ)^{1/3}}{R\sigma_s^{1/3}}, \frac{\sigma_s}{E}, \frac{\gamma}{ER}\ \nu, f, n\right\} \end{aligned} \tag{2.4}$$

plane case

$$R\,\frac{(E\gamma)^{1/3}}{\text{Э}^{2/3}} = f_3\left\{\frac{\text{Э}^{1/2}}{R\sigma_s^{1/2}},\ \frac{\sigma_s}{E},\ \frac{\gamma}{ER},\ \nu,\ f,\ n\right\}$$
$$R\,\frac{(E\gamma)^{1/3}}{(cJ)^{2/3}} = f_4\left\{\frac{(cJ)^{1/2}}{R\sigma_s^{1/2}},\ \frac{\sigma_s}{E},\ \frac{\gamma}{ER},\ \nu,\ f,\ n\right\} \qquad (2.5)$$

Here, σ_S is the characteristic stress of the limiting state in the elastic-plastic model.

In relation to explosions in soils the best known models are the Coulomb-Taylor-Penney [5] and the Mises-Schleicher-Grigoryan [6] models. Hill, Koryavov, Zwolinski, Aliev, and others have used various theories of the limiting state for solving the problem of a spherical explosion.

We recall that for brittle bodies the concept of a limiting state, on which the models of an ideally elastic-plastic body are based, is fundamentally inapplicable; in particular, the fracture stress for a brittle body depends importantly on the structure of the material, which varies during the fracture process.

Hopkinson's rule $R \sim \text{Э}^{1/3}$ is obtained from (2.4) as a limiting case. Another limit of applicability of Hopkinson's rule is associated with the relatively large influence of gravity for more powerful explosions [5]. In practice the "one-third" and "two-fifths" power laws are indistiguishable as a result of the scatter of the data.

It is desirable to analyze certain specific physicotechnical problems leading to the formulation of the problem of a point explosion with determination of the corresponding values of Э or J.

3. Release of chemical energy. In the case of ordinary explosives (TNT, powder, etc.) the fracture mechanism is as follows. As a result of the high-velocity chemical reaction the solid or liquid explosive is converted into a gas at high pressure. The latter is responsible for the destruction and deformation of the body.

In the case in question the total explosion energy Э is directly proportional to the internal energy of the explosive charge, which, in turn, is directly proportional to the mass of the charge Q, i.e.,

$$\text{Э} = \eta Q \qquad (3.1)$$

Here, the proportionality factor η depends on the shape of the cavity, the shape of the charge, the arrangement of the charge in the cavity, the method of initiating the explosion, and the location of the initial cavity (at the surface or remote from the surface). In the case of a surface explosion the dependence of η on these factors is particularly strong.

According to the data presented in [7], 60–70% of all the chemical energy of the charge is converted into mechanical energy, which makes it possible to estimate the value of η.

4. Passage of a wave through a defect. Let the body contain an initial cavity or a defect of the crack type with area S_0; when a powerful short-duration tension wave with a stress of the order of σ passes through the defect, an impulse of the order of $J \approx \sigma S_0 \tau$ acts on each wall of the crack. It is required to determine the final size of the crack R [8]. This problem is of interest in connection with the safety of structures and the stability of slopes.

If $R \gg l_0$, where l_0 is the linear dimension of the initial crack and $l_0 \sim c\tau$, then it makes sense to formulate the point problem with a finite impulse J; in this case R is determined from the second of Eqs. (2.2).

5. Impact on a half-space. In the case of normal impact on a brittle half-space of a point mass m moving at a very high velocity v, in order to determine the size of the fracture zone it is necessary to use the first of Eqs. (2.2); the quantity Э will be equal to the kinetic energy of the particle.

At relatively low, almost quasi-static velocities it is necessary to use the second of Eqs. (2.2); the quantity J will be equal to the momentum of the particle. The latter problem is of special interest in connection with percussive boring [9].

The choice of these solutions is based on the following passages to the limit:

$$m = \varepsilon^2 \to 0, \quad v = 1/\varepsilon \to \infty, \quad mv^2 \to 1, \quad mv \to 0$$
$$m = 1/\varepsilon \to \infty, \quad v = \varepsilon \to 0, \quad mv^2 \to 0, \quad mv \to 1 \tag{5.1}$$

As the formula describing intermediate cases it is natural to take the approximate expression

$$R = \lambda_3 \frac{m^{2/5}}{(E\gamma)^{1/5}} \left[(cv)^{2/5} + \zeta_1 \left(\frac{1}{2} v^2\right)^{2/5}\right] \quad \left(\zeta_1 = \frac{\lambda_1}{\lambda_3}\right) \tag{5.2}$$

In the plane case the problem of the impact of an absolutely rigid wedge against a brittle half-space with an initial crack of length l_0 deserves attention; as a result of cleavage the length of the crack becomes equal to R. In the same way as before, when $R \gg l_0$ it is possible, using (2.3), to find the following approximate formula for R:

$$R = \lambda_4 \frac{m^{2/3}}{(E\gamma)^{1/3}} \left[(cv)^{2/3} + \zeta_2 \left(\frac{1}{2} v^2\right)^{2/3}\right] \quad \left(\zeta_2 = \frac{\lambda_2}{\lambda_4}\right) \tag{5.3}$$

Here, m is the mass of the wedge per unit length. In this case the constants λ_2 and λ_4 also depend on the wedge angle.

6. Release of thermal energy. In the case of underground atomic explosions in hard rocks the atomic energy of the charge W (according to Einstein's formula equal to the product of the square of the speed of light and the mass defect of the nuclear fuel) is converted into thermal and radiation energy. At the center of the explosion a dense high-temperature plasma is formed from the material of the rock; the mechanical destruction of the rock remote from the center proceeds in exactly the same way as in an ordinary chemical explosion only on a larger scale.

In this case the quantity ϑ is directly proportional to W:

$$\vartheta = \xi W \tag{6.1}$$

However, the proportionality factor ξ is, of course, much less than for an ordinary explosive. According to the data presented in [7], 20–30% of the entire energy W is converted into mechanical energy, which makes it possible to estimate the value of ξ. Relation (2.2), together with (6.1), gives a rather good description of the existing data on underground nuclear explosions [7, 10].

In conclusion, the author thanks V. N. Mosinets for discussing his work.

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